A Problem Solving Process

CHEMISTRY TEACHERS' DAYS 2019

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Try this in groups:

- *You are a member of a group of people organising a music festival on the outskirts of Auckland. You are in charge of catering and your first task is to produce a budget with a restricted bottom line. How do you go about doing that?*
- *Share your answer with another group.*
- *Think about the strategies/processes you used to solve the problem*

Problem Solving Processes typically involve:

- Identifying the challenge
- Finding relationships between variables
- Assembling and organising the required information
- Implementing the calculations
- Checking that the answer is within the required limits

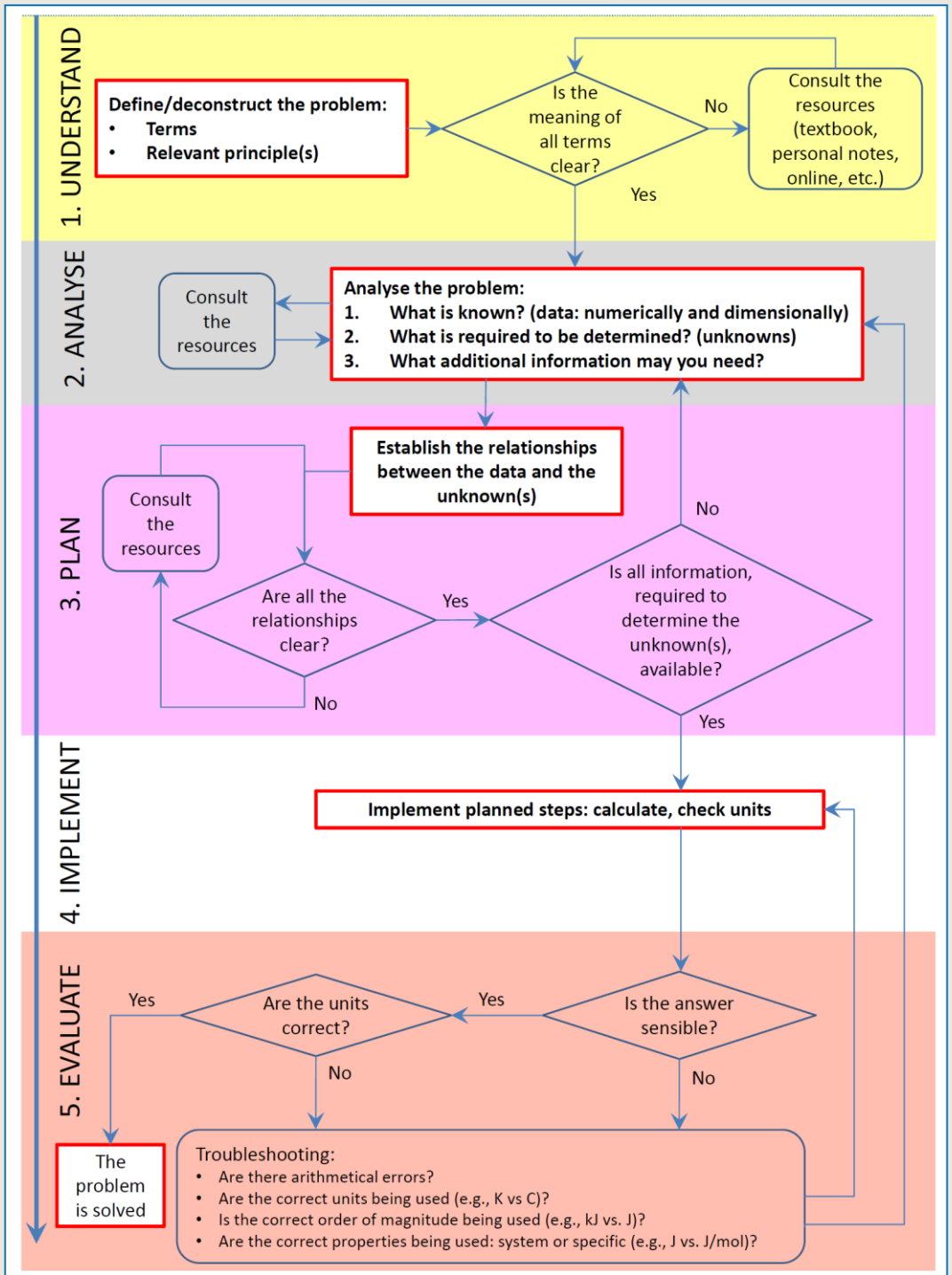
A 'GOLDILOCKS' PROCESS FOR PROBLEM SOLVING

Critical thinking and problem solving are not the same as using previously memorised procedures (although these might be used to solve the problem)

1. **Understand – define/deconstruct the problem**
   What do the terms mean? What question is being asked?
   What chemical principles are involved in this problem? Can you identify key types of chemical reactions (oxidation – reduction, acid – base, precipitation)?
2. **Analyse**
   What is known? What is unknown? What key pieces of information are given? Is some information irrelevant? What else do I need to solve the problem (equations, formulae etc)? Can I show the information in a flow chart?
3. **Plan**
   What are the connections between the known and the unknown?
4. **Implement**
   Carry out the plan - calculations etc
5. **Evaluate**
   Does the answer make sense? (units, order of magnitude etc).
   Did I answer the question(s)?

Yuriev, E., Naidu, S., Schembri, L. S. and Short, J. L. (2017) Scaffolding the development of problem-solving skills in chemistry: guiding novice students out of dead ends and false starts. Chemistry Education Research and Practice, 18, 486-504.

A 'GOLDILOCKS' PROCESS FOR PROBLEM SOLVING



TO SOLVE: Sodium hypochlorite, NaClO, is the active ingredient in many bleaches. Calculate the ratio if the concentrations of ClO^- and HClO in a bleach solution having a pH adjusted to 6.5 by the use of a strong acid. $pK_a(HOCl) = 7.53$

- Do you **understand** what this problem describes?

  $NaClO \rightarrow Na^+ + OCl^-$ $OCl^- + H_2O \rightleftharpoons HOCl + OH^-$

  OCl^- is a weak base. Assume that bleach is a buffer solution.

- **Analyse** what is going on.

  Adding OH^- to solution of OCl^- will increase OCl^- and decrease HOCl. Adding acid will reduce OCl^- and increase HOCl. i.e. decreases base/acid ratio.

What is known? pH = 6.5 $pK_a = 7.53$ ($K_a = 10^{7.53}$) $pH = pK_a + \log \frac{[A]}{[HA]}$ or $H_3O^+ = K_a \times \frac{[HA]}{[A^-]}$

What is to be determined? $[ClO^-]/[HClO]$ ratio *What unknown?* (pK_a)

- **Plan** what to do next

Use $pH = pK_a + \log \frac{[A]}{[HA]}$ or $H_3O^+ = K_a \times \frac{[HA]}{[A^-]}$ $\frac{[HA]}{[A^-]} = \frac{H_3O^+}{K_a}$

- Implement

$$\frac{[A^-]}{[HA]} = \frac{H_3O^+}{K_a} = \frac{10^{6.5}}{10^{7.53}} = 10^{6.5-7.53} = 0.093$$

- **Evaluate**: units?? Units not needed since it is a ratio

TO SOLVE: 21.50 mL of acidified 0.0167 mol L^{-1} potassium permanganate solution, $KMnO_4(aq)$ was needed to completely oxidise 20.00 mL of iron(II) sulfate solution, $FeSO_4(aq)$. The end point of the reaction was observed when the colourless solution changes to the purple colour of the permanganate solution. Calculate the concentration of the iron(II) sulfate.

- Do you **understand** what this problem describes?
  Reaction of $KMnO_4$ with $FeSO_4$. Standard solution of $KMnO_4$.
  To find out concentration of $FeSO_4$

- **Analyse** what is going on.
  Reaction of acidified $KMnO_4$ with $FeSO_4$.
  What is known? $V(KMnO_4)$ = 21.50 mL, $c(KMnO_4)$ = 0.0167 mol L^{-1}
  $V(FeSO_4)$ = 20.00 mL
  What is to be determined? $c(FeSO_4)$ *What is unknown?* Balanced equation

- **Plan** what to do next
  Write the balanced equation for the reaction
  Use *c* and *V* of $KMnO_4$ to calculate moles
  Use the equation to find the moles of $FeSO_4$
  Use $n(FeSO_4)$ and $V(FeSO_4)$ to calculate the concentration of $FeSO_4$

- **Implement**
  $5Fe^{2+} + MnO_4^- + 8H^+ \rightarrow 5Fe^{3+} + Mn^{2+} + 4H_2O$
  $n(MnO_4^-) = c(MnO_4^-) \times V(MnO_4^-) = 0.0167 \text{ mol } L^{-1} \times 0.02150 \text{ L} = 3.591 \times 10^{-4} \text{ mol}$
  $n(Fe^{2+}) = 5 \times n(MnO_4^-) = 5 \times 3.591 \times 10^{-4} \text{ mol} = 0.001795 \text{ mol}$
  $c(Fe^{2+}) = \frac{n(Fe^{2+})}{V(Fe^{2+})} = \frac{0.001795 \text{ mol}}{0.02000 \text{ L}} = 0.0898 \text{ mol } L^{-1}$

- **Evaluate:** units?? Sensible

SCHOLARSHIP 2010:

How does change in pH affect the solubility of thallium(I) sulfide?

$K_s(Tl_2S) = 6 \times 10^{-22}$ $pK_a(HS^-) = 11.96$.

- Do you **understand** what this problem describes?

  Thallium sulfide is a sparingly soluble salt. If pH affects the concentration of one ion then it will change the solubility (common ion effect)

- **Analyse** what is going on.

  $Tl_2S(s) \rightleftharpoons 2Tl^{2+}(aq) + S^{2-}(aq)$ $K_s = 6 \times 10^{-22}$

  $HS^- + H_2O \rightleftharpoons S^{2-}(aq) + H_3O^+(aq)$ $pK_a(HS^-) = 11.96$.

  So S^{2-} is a weak base $S^{2-} + H_2O \rightleftharpoons HS^-(aq) + OH^-(aq)$

- **Plan** what to do next

  Use equilibrium equations and common ion effect/Le Chatelier's principles to discuss changes with pH increase and decrease

- **Implement**

  Decreasing pH: $[OH^-]$ is decreased $H_3O^+(aq) + OH^-(aq) \rightarrow 2H_2O$

  $[S^{2-}]$ is decreased: $S^{2-}(aq) + H_2O(\ell) \rightarrow HS^-(aq) + OH^-(aq)$

  This causes more Tl_2S to dissolve to restore $[S^{2-}]$. $Tl_2S(s) \rightleftharpoons 2Tl^{2+}(aq) + S^{2-}(aq)$

  Therefore the solubility of Tl_2S increases as the pH decreases.

  Increasing pH: Increasing $[OH^-]$ increases $[S^{2-}]$ so more Tl_2S to precipitates to use up $[S^{2-}]$.

- **Evaluate:**

TO SOLVE: The pH inside most cells is maintained at around 7.4 by a phosphate buffer made up of $H_2PO_4^-(aq)$ and its conjugate base HPO_4^{2-}. $pK_a(H_2PO_4^-) = 7.2$.
Typically, total phosphorus concentration in a cell, $[H_2PO_4^- + HPO_4^{2-}]$ is 0.020 mol L^{-1}.
Calculate typical values of $[H_2PO_4^-]$ and $[HPO_4^{2-}]$ inside a cell.

- Do you **understand** what this problem describes?
  Buffer solution, given pH and pK_a. pH depends on $[H_2PO_4^-]/ [HPO_4^{2-}]$ ratio.
  Need to find these concentrations. Need buffer pH equation.

- **Analyse** what is going on.
  pH = 7.4 $pK_a(H_2PO_4^-) = 7.2$ $[H_2PO_4^-] + [HPO_4^{2-}] = 0.020$ mol L^{-1}

  $$pH = pK_a + \log \frac{[A]}{[HA]} \text{ or } H_3O^+ = K_a \times \frac{[HA]}{[A^-]}$$

- **Plan** what to do next
  Use pH equation to calculate ratio of acid and base, then use total concentration to calculate species concentrations

- **Implement** $H_2PO_4^- + H_2O \rightleftharpoons HPO_4^{2-} + H_3O^+$

  $$pH = pK_a + \log \frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = 7.2 + \log \frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = 7.4 \qquad \frac{[HPO_4^{2-}]}{[H_2PO_4^-]} = 1.6$$

  $[H_2PO_4^-(aq)] + [HPO_4^{2-}(aq)] = 0.020$ mol L^{-1} and $[HPO_4^{2-}(aq)] = 1.6 \times [H_2PO_4^-(aq)]$

  So $[H_2PO_4^-(aq)] + 1.6 \times [H_2PO_4^-(aq)] = 0.020$ mol L^{-1} $[H_2PO_4^-(aq)] = 7.7 \times 10^{-3}$ mol L^{-1}

  Then $[HPO_4^{2-}(aq)] = 1.6 \times (7.7 \times 10^{-3}$ mol $L^{-1}) = 1.2 \times 10^{-2}$ mol L^{-1}

- **Evaluate**: Is the answer sensible? Units?

SCHOLARSHIP 2009: Titanium is a Group 4 transition metal. There are three different chlorides of titanium: $TiCl_2$, $TiCl_3$ and $TiCl_4$. One of these chlorides, A, is a solid that dissolves in water to produce a mildly acidic purple solution. On standing in the presence of air, the colour of this solution fades, and a white solid, TiO_2, is formed. The chlorides B and C are very reactive toward water. B is a liquid and reacts to produce a strongly acidic solution and TiO_2. C reacts with acidified water to produce a purple solution and hydrogen gas.

Identify which of the chlorides of titanium is A, which is B and which is C.

Justify your answers using the properties of transition metals, including the colours and reactions outlined above, and / or by analogy with the chemistry of other transition metals. Include balanced equations for the reactions described.

Note: Colour in transition metal compounds occurs if there are partially filled *d* orbitals.

Understand: Transition metals ions - Ti^{2+}, Ti^{3+}, Ti^{4+}

Analyse: Ti is $3d^2 4s^2$, Ti^{2+} is $3d^2$, Ti^{3+} is $3d^1$, Ti^{4+} is $3d$

A: solid, soluble, purple, acidic solution. In air → white TiO_2 (Ti^{4+})

B: liquid, reacts with water → acidic solution + white TiO_2 (Ti^{4+})

C: reacts with water → purple solution + $H_2(g)$

Plan: **A** oxidised to Ti^{4+}, purple – needs *d* electrons

B produces H_3O^+ in water + oxide

C reduces water ($H_2O \rightarrow H_2$) so is oxidised, oxidation product is purple (A)

Implement: **A** and **C** can both be oxidised (lose e^-) so cannot be Ti^{4+}

C is oxidised to **A** so must be Ti^{2+}, **A** is Ti^{3+}, **B** is Ti^{4+}

C → A → B

Evaluate: Need to justify answer and write equations

SCHOLARSHIP 2009 ANSWER

A is $TiCl_3$. Ti^{3+} has 1 *d* electron so this can be promoted to a higher energy *d* orbital by absorbing coloured light. The acidity of Ti(III) is similar to that of Fe(III) in water

$[Ti(H_2O)_6]^{3+} + H_2O \rightarrow [Ti(H_2O)_5OH]^{2+} + H_3O^+$

The reaction with O_2 will be an oxidation and will result in a compound containing Ti(IV). TiO_2 is a white solid.

$4Ti^{3+} + O_2 + 6H_2O \rightarrow 4TiO_2 + 12H^+$

B is $TiCl_4$. $TiCl_4$ reacts with water: $TiCl_4 + 2H_2O \rightarrow TiO_2 + 4HCl$

TiO_2 is a white solid and HCl is a strong acid in water

C is $TiCl_2$. The purple solution on reaction with water suggests that it is oxidised to produce Ti^{3+}, releasing hydrogen as the reduction product.

$2Ti^{2+} + 2H^+ \rightarrow 2Ti^{3+} + H_2$

SCHOLARSHIP 2005

Account for the observation that platinum metal reacts with aqua regia (a mixture of hydrochloric and nitric acids) but not with concentrated hydrochloric or nitric acid individually.

$Pt^{2+} + 2e^- \rightarrow Pt$ $\quad E° = 1.18$ V

$NO_3^- + 4H^+ + 3e^- \rightarrow NO + 2H_2O$ $\quad E° = 0.96$ V

$PtCl_4^{2-} + 2e^- \rightarrow Pt + 4Cl^-$ $\quad E° = 0.76$ V

$PtCl_6^{2-} + 2e^- \rightarrow PtCl_4^{2-} + 2Cl^-$ $\quad E° = 0.68$ V

$2H^+ + 2e^- \rightarrow H_2$ $\quad E° = 0.00$ V

Understand: Pt + HCl no reaction, Pt + HNO_3 nor reaction, Pt + HCl + HNO_3 reacts

Analyse: For a reaction to occur $E°_{cell} > 0$

So expect that Pt + H^+ + Cl^- and Pt + H^+ + NO_3^- $E°_{cell} < 0$ but Pt + H^+ + Cl^- + NO_3^- $E°_{cell} > 0$

Plan:

Use the ½ equations to write balanced equations for each of the reactions and calculate $E°_{cell}$

Implement:

- $Pt + 2H^+ \rightarrow Pt^{2+} + H_2$ $\quad E_{rxn} = E_{red} - E_{ox} = 0.00 - 1.18$ V

  $3Pt + 2NO_3^- + 8H^+ \rightarrow 3Pt^{2+} + 2NO + 6H_2O$ $\quad E_{rxn} = E_{red} - E_{ox} = 0.96 - 1.18$ V

  $Pt + 4Cl^- + 2H^+ \rightarrow H_2 + PtCl_4^{2-}$ $\quad E_{rxn} = E_{red} - E_{ox} = 0.00 - 0.76$ V

  All $E°_{cell} < 0$ so none are spontaneous

- $2NO_3^- + 3Pt + 12Cl^- + 8H^+ \rightarrow 3PtCl_4^{2-} + 2NO + 4H_2O$ $\quad E_{rxn} = E_{red} - E_{ox} = 0.96 - 0.76$ V

  $E°_{cell} > 0$ so reaction goes

Evaluate: Need to justify answer and write equations

$2NO_3^- + 3PtCl_4^{2-} + 6Cl^- + 8H^+ \rightarrow 3PtCl_6^{2-} + 2NO + 4H_2O$ $\quad E_{rxn} = E_{red} - E_{ox} = 0.96 - 0.68$ V > 0